Acta Technica 62 (2017), No. 5B, 139–148 © 2017 Institute of Thermomechanics CAS, v.v.i.

Relationship prioritized Pythagorean fuzzy aggregation operators and their application on networks

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Abstract. PROMTHEE has arouse many study of MADM problems based on the situation which there exists a prioritization of criteria. This outranking relation can solve part difficulties, while other remains. When the relationship between attributes deeply affect the performance of alternatives, we need to seek for other approach from the contrary direction, which we call relationship prioritized methods. To realize this purpose, we utilize the mathematical tool of fuzzy measure which can clearly depict influence of attributes relation. Meanwhile, we invent two Pythagorean fuzzy operators based on fuzzy measure, RP-PFOWA and RP-PFOWG operators to accomplish the goal. Additionally, an example of network selection is given to illustrate the validity of the operators.

Key words. PROMETHEE, relationship prioritized, pythagorean fuzzy set, fuzzy measure, network selection.

1. Introduction

The idea of PROMETHEE is firstly proposed by Brans and Vincke in 1982[1]. The main thought of using PROMETHEE method into MADM problem is that, admitting the existence of a prioritization of criteria. This outranking relation can be realized by distributing different weights according to the priority the attributes belong to. The benefits of this method lies on that, without normalized process, there will be no risk of information deviation. Yager [2,3] provided the model to

[1]Acknowledgement - This work has been partly supported by the National Natural Science Foundation of China [grant numbers 61702543, 61271254, 71501186, 71401176], the Natural Science Foundation of Jiangsu Province of China [grant number BK20141071, BK20140065], the 333 high-level talent training project of Jiangsu Province of China [grant number BRA 2016542].

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apply this method into fuzzy sets. Yu and Xu expanded it into intuitionistic fuzzy area.

However, in practice, we may have the circumstance that the relationship between attributes means more than the performance of a single attribute. For example, teacher select student to attend knowledge competition. The best alternative shouldn't be the one who has the best score on single subject, but the one who has good performance on all subjects, even though not the best for each. At this point, a new concept of priority should be invented which is relationship prioritized method. Aiming to achieve this purpose, we utilize the tool of fuzzy measure [12] which can be used to illustrate the interaction of considered attributes. This kind of application [13-18] in fuzzy aggregation operators is not an emerging idea. However, the Pythagorean fuzzy set has been seldom put into consideration.

Pythagorean fuzzy set (PFS) is firstly proposed by Yager [4-7], which is an extension of intuitionistic fuzzy set (IFS) [7-9]. The membership and non-membership degree of PFS contains more than IFS according to their definition. Therefore, PFS can express subtler uncertainties than IFS, which ia the reason why we choose it as our searching environment.

This paper is aim to fulfill the achievement of developing an inverse train of thought than PROMETHEE, acquiring a relationship prioritized aggregation operator based on Pythagorean fuzzy environment. After that, the operators are applied in aggregating information for multi-criteria decision making examples on network selection.

2. Basic knowledge review

We firstly recall the Pythagorean fuzzy set (PFS) and Pythagorean fuzzy number (PFN), as well as their definition, operations and properties. Then, we introduce the concept of fuzzy measure.

Definition 1 [4-7,10] A PFS P is a set which meets the following form, where set S is the universe of discourse under consideration

$$P = \{\langle s, P(\mu_p(s), \nu_p(s))\rangle \,|s \in S\}. \tag{1}$$

in which$\mu_p : S \to [0,1]$represents the membership degree ,$\nu_p : S \to [0,1]$defines the non-membership degree of the element$s \in S$to$P$. Respectively, for every $s \in S$, it respects to the principle$0 \leq (\mu_p(s))^2 + (\nu_p(s))^2 \leq 1$. $\pi_p(s) = \sqrt{1 - \mu_p^2(s) - \nu_p^2(s)}$ is the degree of uncertainty$s$ to P. Pythagorean fuzzy number (PFN) $\beta = P(\mu_\beta, \nu_\beta)$is short for$P(\mu_p(s), \nu_p(s))$, where$\mu_\beta, \nu_\beta \in [0,1]$,$\pi_\beta$=$\sqrt{1-(\mu_\beta)^2-(\nu_\beta)^2}$,and$(\mu_\beta)^2+(\nu_\beta)^2 \leq 1$.

Definition 2 [10] For three PFNs$\beta = P(\mu_\beta, \nu_\beta)$,$\beta_1 = P(\mu_{\beta_1}, \nu_{\beta_1})$, $\beta_2 = P(\mu_{\beta_2}, \nu_{\beta_2})$, they have the operations rules as follow:

1. $\beta_1 \oplus \beta_2$=$P\left(\sqrt{\mu_{\beta_1}^2 + \mu_{\beta_2}^2 - \mu_{\beta_1}^2\mu_{\beta_2}^2}, \nu_{\beta_1}\nu_{\beta_2}\right)$.

2. $\beta_1 \oplus \beta_2 = P\left(\sqrt{\mu_{\beta_1}^2 + \mu_{\beta_2}^2 - \mu_{\beta_1}^2 \mu_{\beta_2}^2}, \nu_{\beta_1}\nu_{\beta_2}\right)$.

3. $\lambda\beta = P\left(\sqrt{1-\left(1-\mu_\beta^2\right)^\lambda}, (\nu_\beta)^\lambda\right), \lambda > 0.$

4. $\beta^\lambda = P\left((\mu_\beta)^\lambda, \sqrt{1-\left(1-\nu_\beta^2\right)^\lambda}\right), \lambda > 0.$

Definition 3 [10] The score function of PFN β is defined as:

$$S_c(\beta) = (\mu_\beta)^2 - (\nu_\beta)^2. \tag{2}$$

Definition 4 [11] For any PFN$\beta = P(\mu_\beta, \nu_\beta)$, the accuracy function ofβcan be defined as follows:

$$a(\beta) = (\mu_\beta)^2 + (\nu_\beta)^2, \tag{3}$$

Where$a(\beta) \in [0, 1]$.

Set$\beta_j = P\left(\mu_{\beta_j}, \nu_{\beta_j}\right), (j = 1, 2)$to be two PFNs, $S_c(\beta_1)$and $S_c(\beta_2)$ are the scores of PFNs $\beta_1$and β_2, $a(\beta_1)$and $a(\beta_2)$are the accuracy of PFNsβ_1,β_2. Then

1. When $S_c(\beta_1) < S_c(\beta_2)$, then$\beta_1 < \beta_2$;
2. When $S_c(\beta_1) > S_c(\beta_2)$, then$\beta_1 > \beta_2$;
3. When $S_c(\beta_1) = S_c(\beta_2)$, then
4. When $a(\beta_1) > a(\beta_2)$, then$\beta_1 > \beta_2$;
5. When $a(\beta_1) < a(\beta_2)$, then$\beta_1 < \beta_2$;
6. When $a(\beta_1) = a(\beta_2)$, then$\beta_1 \sim \beta_2$.

Definition 5 [19-21] Let$F_m$be the fuzzy measure on a finite space$S$, which is a mapping$F_m : \vartheta[S] \to [0, 1]$ satisfying conditions:

1. $F_m[\emptyset] = 0, F_m[S] = 1$;
2. $A, B \subseteq S$and$A \subseteq B$, $F_m(A) \leq F_m(B)$;
3. $F_m(A \cup B) = F_m(A) + F_m(B) + \beta F_m(A) F_m(B)$, for all$A, B \subseteq S$, and$A \cap B = \emptyset$, where $\beta \in (-1, \infty)$.

Some explanation of the parameter$\beta$are as blow:

When$\beta = 0$, condition (3) in Def.2.4 is simplified to $F_m(A \cup B) = F_m(A) + F_m(B)$, and the fuzzy measure reduce to an addictive measure, which is shown as:

$$F_m(A) = \sum_{s_i \in A} F_m(\{s_i\}). \tag{4}$$

In practice, it signifies the attributes under consideration is dependent.

When$\beta > 0$, the condition shows that, there is mutual promotion between the attributes. The greater the value of β is, the stronger the interaction is between the two properties.

When$\beta < 0$, contrary to the above situation, the attributes weaken each other when being aggregated. And the greater the value of β is, the stronger the weaken effect is. From the above definition, we can see that the value of β can be used as a measure of the relationship between several attributes for multiple attribute decision making. Here we will describe how to determine the parametersβ. According to the definition in [19], when $A$is a subset of$S$, we have:

$$\begin{array}{l} F_m\left(S\right) = F_m\left(\bigcup_{i=1}^{n} s_i\right) \\ = \left\{ \begin{array}{c} \frac{1}{\rho}\left(\prod_{i=1}^{n}\left(1+\rho F_m\left(s_i\right)\right)-1\right), \rho \neq 0 \\ \sum_{i=1}^{n} F_m\left(s_i\right), \rho = 0 \end{array} \right. \end{array} . \tag{5}$$

$$\begin{array}{l} F_m\left(A\right) = \\ \left\{ \begin{array}{c} \frac{1}{\rho}\left(\prod_{s_i \in A}\left(1+\rho F_m\left(s_i\right)\right)-1\right), \rho \neq 0 \\ \sum_{x_i \in A} F_m\left(s_i\right), \rho = 0 \end{array} \right. \end{array} . \tag{6}$$

Then, we can further get the unique value of β, which is shown as:

$$\begin{array}{l} F_m\left(S\right) = 1, \\ \rho{+}1{=}\prod_{i=1}^{n}\left(1+\rho F_m\left(s_i\right)\right). \end{array} \tag{7}$$

After acquiringβ, we will further aware of the fuzzy measure of subsets of S, depending on equation (6). We can get the weight information related to the interaction between attributes, rather than the traditional value of the weight information according to the priority of attributes. Therefore, we will employ it for the Pythagorean fuzzy set of information integration, getting brand new integrated operators in order to gain more scientific description of the actual situation. In this way, the accurate integration values to will lead to more correct decisions

3. Relationship prioritized operators

Definition 6 Relationship prioritized Pythagorean fuzzy order weighted averaging (RF-PFOWA) operator.

S is a collection of PFNs, $S\left\{\beta_i = P(\mu_{\beta_i}, \nu_{\beta_i})\right\}$, where$i = 1, 2, ..., n$. $s_i$has the ith largest value in β_i, and set$L_i$be a subset of S??it meets the following requirements:

$$\left\{ \begin{array}{c} L_0{=}\emptyset \\ L_i{=}\left\{\sum_{k=1}^{i} s_k\right\} \end{array} \right. .$$

The Relationship prioritized Pythagorean fuzzy order weighted averaging operator is defined as follow:

$$\begin{aligned} &RF - PFOWA(\beta_1, \beta_2, \dots, \beta_n) \\ &= \oplus_{i=1}^{n}(F_m(L_i) - F_m(L_{i-1}))s_i, \end{aligned} \tag{8}$$

$F_m$is the fuzzy measure on$S$, according to the operations of PFNs in Def.2, the PFCOWA operator has form as below:

$$\begin{aligned} &RF - PFOWA(\beta_1, \beta_2, \dots, \beta_n) \\ &= \oplus_{i=1}^{n}(F_m(L_i) - F_m(L_{i-1}))s_i \\ &= \mathrm{P}\left(\sqrt{1 - \prod_{i=1}^{n}\left(1 - \mu_{s_i}^2\right)^{(F_m(L_i) - F_m(L_{i-1}))}}, \prod_{i=1}^{n}\nu_{s_i}^{(F_m(L_i) - F_m(L_{i-1}))}\right). \end{aligned} \tag{9}$$

The result of aggregation is still PFNs.

Definition 7 Relationship prioritized Pythagorean fuzzy order weighted geometric (RF-PFOWG) operator.

$S$is a collection of PFNs, $S\{\beta_i = P(\mu_{\beta_i}, \nu_{\beta_i})\}$, where$i = 1, 2, ..., n$. $s_i$has the ith largest value inβ_i, and set$L_i$be a subset of S,it meets the following requirements:

$$\left\{ \begin{array}{c} L_0 = \emptyset \\ L_i = \left\{\sum_{k=1}^{i} s_k\right\} \end{array} \right. .$$

The Relationship prioritized Pythagorean fuzzy order weighted geometric operator is defined as follow:

$$\begin{aligned} &RP - PFOWG(\beta_1, \beta_2, \dots, \beta_n) \\ &= \otimes_{i=1}^{n} s_i^{(F_m(L_i) - F_m(L_{i-1}))}, \end{aligned} \tag{10}$$

$F_m$is the fuzzy measure on$S$, according to the operations of PFNs in Def.2.2, the PFCOWA operator has form as below:

$$\begin{aligned} &RP - PFOWG(\beta_1, \beta_2, \dots, \beta_n) = \otimes_{i=1}^{n} x_i^{(F_m(L_i) - F_m(L_{i-1}))} \\ &= \mathrm{P}\left(\prod_{i=1}^{n}\mu_{s_i}^{(F_m(L_i) - F_m(L_{i-1}))}, \sqrt{1 - \prod_{i=1}^{n}\left(1 - \nu_{s_i}^2\right)^{(F_m(L_i) - F_m(L_{i-1}))}}\right). \end{aligned} \tag{11}$$

The result of aggregation is still PFNs. It can be easily proved that RP-PFOWG has the same properties with RP-PFOWA.

4. Network application

Consider four kinds of network business: session, stream, interactive, background under four different type network: UMTS, 802.11.a, 802.11.b, WIMAX. According to the characteristics of each business, respectively, they have different requirements towards different attributes. For each business, they focus on different attributes of network. The session business (such as voice, video phone) is more sensitive for real-time indicators like delay, jitter, but the requirement on packet loss rate is not high. The flow, interactive and background businesses are sensitive on packet loss rate and bit error while more tolerant on time delay. We take session business as an example to illustrate the calculate process and observe the decision making results.

Firstly, we take$N_i(i = 1, 2, 3, 4)$as the set of network, in which N_1, N_2, N_3, N_4 respectively represents for network UMTS, 802.11.a, 802.11.b, WIMAX. $Q_j(j = 1, 2, 3, 4, 5)$is the set of attributes under consideration, in whichQ_1, Q_2, Q_3, Q_4, Q_5respectively stands for attribute of delay, jitter, throughput, packet loss rate and cost. According to the original data in [22], the Pythagorean fuzzy judgement matrix $\{\beta_{ij}\}$is illustrated in Table 1. And $\beta_{ij}$is an Pythagorean fuzzy number which indicates the degree that network$N_i$satisfies the requirement of attribute $Q_j$about the business under discussion.

Table 1. The judgement matrix of session

β_{ij}	Q_1	Q_2	Q_3	Q_4	Q_5
N_1	(0.71, 0.33)	(0.73, 0.31)	(0.90, 0.27)	(0.74, 0.32)	(0.18, 0.84)
N_2	(0.28, 0.77)	(0.24, 0.82)	(0.79, 0.32)	(0.34, 0.62)	(0.73, 0.32)
N_3	(0.28, 0.77)	(0.24, 0.82)	(0.72, 0.38)	(0.34, 0.62)	(0.73, 0.32)
N_4	(0.43, 0.65)	(0.56, 0.62)	(0.73, 0.36)	(0.45, 0.68)	(0.39, 0.78)

Then, considering different business concerns about network performance distinctively, and there exists the possible interaction between these properties. We assume the fuzzy measure of different attributes and we will get ρ from equation (7). $F_m(\emptyset) = 0$,$F_m(Q_1) = 0.4$,$F_m(Q_2) = 0.4$,$F_m(Q_3) = 0.2$,$F_m(Q_4) = 0.1$,$F_m(Q_5) = 0.3$,$\rho = -0.6253$. Other fuzzy measure can be calculated by equation (6).

According to the above data conditions, we use RP-PFOWA, RP-PFWOG operators to integrate the attributes information. After comparing the integrated data, we can form a decision strategy. Still, we take the session business environment as an example to elaborate the process of data integration and comparison in detail.

By using equation (6), (7), (9), (10) and (11), we get the aggregated results which can be seen in table 2. After calculate the score function of the results, we get figure 1 which has sharp contrast between alternatives. Finally, we list the ranking results of all the alternatives as well as the decision strategy provided by operators for the session business, which is clearly shown in table 3. In reference [22], the recommended strategy of session business is UMITS > WiMAX > 802.11a > 802.11b

Table 2. Aggregated results

session	RP-PFOWA	RP-PFOWG
UMTS	(0.75051 0.12425)	(0.3889 0.47469)
802.11a	(0.53077 0.31589)	(0.1528 0.65733)
802.11b	(0.49781 0.32553)	(0.14769 0.65913)
WiMAX	(0.53067 0.35575)	(0.2382 0.64267)

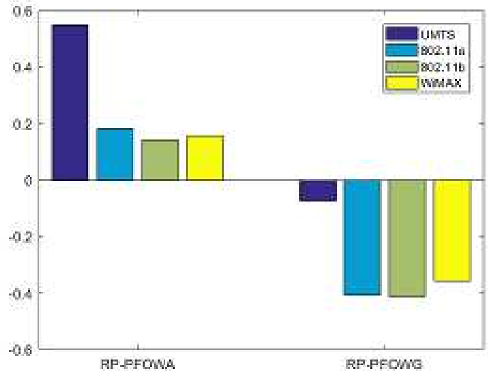

Fig. 1. Comparison of aggregated results

Table 3. Decision strategies by aggregation results

Ranking	1	2	3	4
RP-PFOWA	UMTS	802.11a	WiMAX	802.11b
RP-PFOWG	UMTS	WiMAX	802.11a	802.11b

As we can see from table 3, two operator have tiny difference with each other. The difference comes from different calculation process of averaging and geometric which should be decided by the character of attributes. Relation of attributes contribute to the final results compared with judgment by single attribute. The final results accord with the practical strategy which prove validity of the operators.

5. Conclusion

We bring in fuzzy measure to information aggregating process in Pythagorean fuzzy information, generating two fundamental operators, including RP-PFOWA, RP-PFOWG. The common character of them is the ability to express interconnection between the attributes by weighted variables which make the relationship as a priority when making decision. Furthermore, an example of application is given. We use the developed operators to aggregate attributes information for decision making and compare the results with practical strategy which verify the correctness of this method. In the future, further study will be put on some extension operator of relationship prioritized principle, and sufficiently demonstration of their practical application area.

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Received November 16, 2017